The cosmic pint: estimates and extra detail

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This a summary of some of the rough calculations and the literature I used for my Pint of Science talk to estimate the number of various things that you can find in 1 (UK) pint ($\sim$0.00057 m^3). Because this is cosmology the estimates are quite rough, usually answers within a factor of 10 are okay. Some of you I'm sure may be horrified by my aggressive rounding but this is all fine in the scheme of things, I promise. This little document is written to be understandable for someone with a little bit of knowledge of maths and a general interest in physics. If anything doesn't make sense or seems strange, please let me know.

1. THE LOCAL CLOUD

The local interstellar medium is well measured, as in for example Ref. [1]. The density is very low, around 0.1 atoms per cm^3, or about 50 per pint. This is mostly neutral hydrogen and helium, with a small contribution from cosmic dust particles, simple molecules as well as heavier elements which are produced in stars. Interestingly our local cloud is slightly less dense than the Milky Way average which is about 0.5 per cm^3. This could be because a supernova cleared out the local region before the formation of the Sun.

2. COSMIC RAYS

I spoke about the number of cosmic rays going through your pint glass every second. Most of these get stopped by the atmosphere and the magnetic field of the Earth but if the Earth is not there then there are a huge number of them around flying through space. Most cosmic rays come from processes happening inside stars and other astronomical bodies. They can have extremely high energies, in fact the highest energy particles we have ever seen in nature are cosmic rays. They can have energies thousands of times greater than we can achieve, even in our largest particle accelerator, the large hadron collider. The flux of cosmic rays is well measured by the AMS experiment on the international space station (amongst many others). Most cosmic rays are protons, alpha particles and electrons, with a small number of positrons, antiprotons and heavier nuclei. Ref. [2] have a nice summary of the measurements of the cosmic ray fluxes, the total flux is around 1×10^4/m^2/s. We can convert this into a number by multiplying by the cross sectional area of a pint glass. Let's say 15cm $\times$ 8cm = 120 cm^2. Which means that around 120 cosmic ray alpha particles are flying through the pint glass every second.

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3. DARK MATTER

This is where things get interesting, and quite a lot more subtle. The point about dark matter is that everything we know about it comes from its gravitational influence over objects we can see. Many people have estimated the amount of dark matter in the past from looking at how the stars around us are moving as well as the dynamics of the Milky Way as a whole. The best estimate at the moment is a number for the *mass density* at the location of the Solar System, i.e. the mass of dark matter per pint if you like. This number is around $\rho = 0.3$ GeV/cm^3. GeV stands for giga-electronvolt (10^9 eV) and is the favourite unit of the particle physicist to describe the masses of particles (although strictly it is the rest mass energy rather than just the mass).

So to get the *number* of dark matter particles inside a pint we need to know the mass of the particle. Unfortunately, because we have not detected dark matter, we don't know what this is. However the theories behind the different dark matter candidates give us a rough idea of how heavy they should be.

WIMPs (Weakly Interacting Massive Particles): A rough estimate for how massive a WIMP can be is around 100 GeV. In principle it could be anything from as small as 1 GeV to over 1000 GeV, but 100 GeV is a good benchmark that people often use. Dividing the dark matter density by the mass of the WIMP tells us that there are around 1.7 WIMPs per pint (but really you should keep factors of 10 either way in the back of your mind). Now we want to know how often they interact. The formula for this is quite simple [3],

$$R = NnA^2\sigma v \tag{1}$$

Where N is the number of atoms in the detector (the pint), $n \sim 3000$ m^{-3} is the number density of WIMPs (that we just calculated), A is the number of protons and neutrons in each atom of the detector material, $\sigma \sim 10^{-43}$ cm^2 is called the interaction cross section, a number that tells you how likely it is for the WIMP to interact (more on this in a second)[1]. Finally $v \sim 300$ km/s is the average speed of the WIMP in the Galaxy. Beer is basically water, and the WIMP will mostly only interact with the oxygen atoms because they have bigger nuclei than hydrogen atoms. So we put in $A \sim 16$ for water. A pint of water is about 570 g, which is around $570\,\text{g}/16 = 36$ moles. Using Avogadro's number this tells us that in a pint of beer there are roughly,

$$N = (36\,\text{mols}) \times (6.02 \times 10^{23}) = 2.1 \times 10^{25}\,\text{oxygen atoms}. \tag{2}$$

Plugging the numbers in we get around $R = 0.001$ per year. So I said that WIMPs will interact with a pint of beer around once every 1000 years. However the number that we plugged in for the cross section ($\sigma \sim 10^{-43}$ cm^2) believe it or not is as big as it can be and still not have shown up in our experiments. In principle this number could go all the way down to, say 10^{-60} cm^2 or something (even down to 0 if we are extremely unlucky). So take this calculation with multiple grains of salt, it looks as though dark matter will interact even less frequently than once every 1000 years.

The Large Underground Xenon experiment (LUX) has around 370 kg of Xenon instead of beer. I claimed that this is equivalent to about 5000 pints of beer. The reason

[1] If there are any experts reading this: I picked this number so that the rate matches the proper calculation when integrating over the *differential* cross section, it's equivalent to a WIMP-proton cross section of about 10^{-45} cm^2, around the LUX limit at 100 GeV.

it is equivalent to 5000 pints and not 370 kg /0.57 kg = 650 pints as you might expect is because of the factor of A^2 in the formula above. Because of the way that dark matter interacts more with bigger nuclei, the particle physics ends up enhancing the rate by an extra factor of A over simply the number of particles in the detector. Basically this means that Xenon (which has 131 nucleons in its nucleus) is a better detector material than beer (which has 16). The factor by which it is better is 131/16 = 8, which means that the equivalent mass is about 650 pints multiplied by 8, which is the 5000 number that I quoted.

Axions: For the axions it's a bit harder but we know that the axion gives a power less than 10^{-22} W in the axion dark matter experiment (ADMX [4]) and this is the number I used. The intensity of a $P = 30$ W bulb seen from Mars which is an average distance $d = 225$ million km away, we get by calculating,

$$I = \frac{P}{4\pi d^2} = 4.7 \times 10^{-23}\,\mathrm{W\,m^{-2}} \tag{3}$$

The size of the resonator in ADMX is about 1 square metre. So the the axion power is very roughly equivalent to this.

4. BIG BANG RELICS

The CMB is a black body spectrum with a temperature of $T_{\mathrm{CMB}} = 2.728$ K. The Stefan-Boltzmann law for the energy density of black body radiation is,

$$U_{CMB} = \frac{4\sigma}{c} T_{\mathrm{CMB}}^4 = 4.19 \times 10^{-14}\,\mathrm{J\,m^{-3}} \tag{4}$$

where $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} is the Stefan-Boltzmann constant. The average energy of the black body photons is around $3k_B T_{\mathrm{CMB}} = 1.13 \times 10^{-22}$ J, where $k_B = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant. So the number density of photons left over from the big bang is around 3.7×10^8 per m^3. This means that there are something like 210,000 photons in your pint left over from the Big Bang.

The density of relic neutrinos can be calculated using the physics of neutrino production and the hot Big Bang. The details of which can be found in, for example, Ref. [5]. The calculation leaves a density today of around 113 cm^{-3} per neutrino flavour. Since there are three flavours of neutrinos this means there are around 190,000 in your pint, a very similar number to the number of photons.

5. DARK ENERGY

To estimate the amount of dark energy in a pint we need to first figure out the density of dark energy. We know what the average density of the Universe is, it is a number known as the *critical density* and it has the formula,

$$\rho_c = \frac{3H_0^2}{8\pi G} \tag{5}$$

where $H_0 \simeq 70$ km s^{-1} Mpc^{-1}, is the Hubble parameter today, and $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2} is Newton's constant. The funny units of H_0 we can sort out just by dividing it by the definition of a Megaparsec: Mpc $= 3.1 \times 10^{19}$ km, leaving it in units of s^{-1}. The critical density then evaluates to be about 9.2×10^{-27} kg/m^3. From the Planck satellite observations of the cosmic microwave background we know that dark energy in the form of a cosmological constant, Λ, makes up about 68.3% of this so $\rho_\Lambda = 0.683 * 9.2 \times 10^{-27} = 6.2 \times 10^{-27}$ kg/m^3. We can convert this mass density into an energy density by multiplying by the speed of light squared (because $E = mc^2$) giving us 5.58×10^{-10} Joules/m^3. Then after converting to pints, ultimately leaves us with an answer of 3.17×10^{-13} Joules/pint.

For comparison, 3.17×10^{-13} Joules corresponds to the sound intensity of 2.4×10^{-9} W/m^2, if listened to for 1 second (this is using the fact that each eardrum is about 65 mm^2 in area). In decibels this is $10 \log_{10}(2.4 \times 10^{-9}/10^{-12}) = 34$ dB, which is very quiet, about as loud as a whisper from a few feet away.

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